A Relativization Perspective on Meta-Complexity

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Meta-complexity: "complexity of complexity"

**MCSP** (minimum circuit size problem)

*Input*: a truth table \( \Phi \in \{0,1\}^{2^n} \), representing a function \( f: \{0,1\}^n \rightarrow \{0,1\}^n \)

*Output*: the circuit complexity of \( f \)

Why study meta-complexity?

* Interesting.
* Mysterious.

- Is MCSP NP-hard?
- Are MCSP[2^{n/2}] and MCSP[2^{4n/14}] even related?

* Connections to other areas.
  - Learning
  - Circuit complexity
  - Average-case complexity
  - Cryptography
  - [Hir18]
  - [LP20]

Our results:

- Relativization barriers in meta-complexity
- Big open questions about MCSP cannot be answered in a relativizing way!

In contrast, many recent breakthroughs are indeed relativizing [Hir18, Hir20, Hir21, LP20]

For example, we present a relativized world where MCSP[2^{n/2}] is easy but MCSP[2^{4n/14}] is hard.

Further directions: non-relativizing techniques in meta-complexity?