A Relativization Perspective on Meta-Complexity

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Meta-complexity: "complexity of complexity"

MCSP (minimum circuit size problem)

Input: a truth table $tt \in \{0,1\}^{2^n}$, representing a function $f: \{0,1\}^n \to \{0,1\}$

output: the circuit complexity of f.

Why study meta-complexity?

* interesting.
* mysterious.
  - Is MCSP NP-hard?
  - Are $MCSP[2^{n/2}]$ and $MCSP[2^{n/4}]$ even related?

* connections to other areas.
  - average-case complexity [Hir'18]
  - cryptography [LP'20]
  - learning
  - circuit complexity
  - etc.

Our results: relativization barriers in meta-complexity

Big open questions about MCSP cannot be answered in a relativizing way!

In contrast, many recent breakthroughs are indeed relativizing [Hir'18, Hir'20*, Hir'21*, LP'20]

* modulo a PRG in [BFP'05]

For example, we present a relativized world where $MCSP[2^{n/2}]$ is easy but $MCSP[2^{n/4}]$ is hard.

Further direction: non-relativizing techniques in meta-complexity?